F-x eigen noise suppression

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Summary

F-x eigen filtering performs random noise suppression on 2-D seismic data using eigenimage analysis along constant-frequency slices. The method works equally well on flat or dipping events. Statistical measures of noise suppression performance on artificial data, as well as tests on real data, shows the method to be competitive with prediction-based noise attenuators.

Methodology

Suppose we have a series of *n* equally spaced traces representing, say, a portion of a 2-D stacked section. A typical value for *n* might be 25. For a given temporal frequency extract the complex DFT value for every trace

$$t_1 \; t_2 \; \ldots \; t_n \, .$$

Canales (1984) showed that for noiseless data made up of at most *k* dips the above sequence can be written as the sum of *k* sinusoids

$$t_p = \sum_{i=1}^{k} a_i \, e^{jpb_i}$$

where a_i are complex values and b_i are real values. Form the data matrix

$$\mathbf{A} = \begin{bmatrix} t_1 & t_2 & t_3 & \cdots & t_m \\ t_2 & t_3 & t_4 & \cdots & t_{m+1} \\ t_3 & t_4 & t_5 & \cdots & t_{m+2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ t_{n-m+1} & t_{n-m+2} & t_{n-m+3} & \cdots & t_n \end{bmatrix}.$$

This matrix has Hankel structure, meaning the matrix elements are constant along the anti-diagonals. By adjusting *m* we can change the matrix dimensions. The accepted strategy is to make the matrix as square as possible by choosing *m = n/2*.

The noise suppression method replaces $\mathbf{A}$ with a reduced-rank approximation $F_k(\mathbf{A})$ (Trickett, 2002.) To briefly summarize, begin by taking the singular value decomposition (SVD)

$$\mathbf{A} = \mathbf{U \Sigma V}^H$$

where $\mathbf{U}$ and $\mathbf{V}$ are unitary matrices, and $\mathbf{\Sigma}$ is a real diagonal matrix whose diagonal elements σ_i are ordered such that

$$\sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_n \geq 0.$$

For some small value of *k* form the partial sum

$$F_k(\mathbf{A}) = \mathbf{I}_1 + \mathbf{I}_2 + \cdots + \mathbf{I}_k, \quad \mathbf{I}_i = \sigma_i \mathbf{u}_i \mathbf{v}_i^H$$

where vectors $\mathbf{u}_i$ and $\mathbf{v}_i$ are the *i*th columns of $\mathbf{U}$ and $\mathbf{V}$, respectively.

It can be shown that $F_k(\mathbf{A})$ is the nearest rank-*k* matrix to $\mathbf{A}$, but it is not generally Hankel. We recover the Hankel structure by averaging the elements of each anti-diagonal of $F_k(\mathbf{A})$. This is not optimum, however, and calculating the nearest rank-*k* Hankel matrix to $\mathbf{A}$ is a subject of active research (Park et al, 1999). It is an open question whether doing so would result in a significantly improved noise suppression.

Putting all of this together, the method, which I will call *f-x eigen filtering*, is:

> Take the DFT of each trace.
>
> For each frequency…
>
> > Form the complex-valued Hankel matrix $\mathbf{A}$ from the DFT value of each trace.
> >
> > Calculate the partial sum $F_k(\mathbf{A})$ for some small value of *k*.
> >
> > Average along the anti-diagonals of $F_k(\mathbf{A})$ to recover the Hankel structure. Call this matrix $H_k(\mathbf{A})$.
> >
> > Replace the trace DFT values with the $H_k(\mathbf{A})$ values.
>
> Take the inverse DFT of each trace.

This method was first suggested by Cadzow (1988) for spectral analysis problems, and is well known to the Nuclear Magnetic Resonance (NMR) community. Cadzow actually recommended iterating between the calculation of the partial sum and averaging steps. I have dispensed with it here because it increases the cost without delivering any apparent improvement for this application.

Other f-x noise suppression schemes also use eigen methods. Two examples are spectral matrix filtering (Gounan et al, 1998), which is a frequency-domain version of the K-L transform, and the improvements to f-x prediction filtering suggested by Harris and White (1997). Unlike these, however, our method does not work from a covariance matrix, nor does it extract a prediction operator. More similar to our method is the eigenimage noise suppression proposed by Ulrych et al (1988.) Although not described in detail, it is (apparently) in the t-x domain and probably does not give satisfactory results for dipping data.

The amount of attenuated noise can be increased by increasing *n* and most importantly decreasing *k*. By doing so, however, we also increase the chance of distorting the coherent signal. In practise typical values of *k* are 1 (harsh), 2 (strong), and 3 (moderate).

Exactness Property: *If a noiseless seismic section contains no more than k dips then* $H_k(\mathbf{A}) = \mathbf{A}$.

Proof: Stephenson (1988).

In other words, eigen filtering does nothing to noiseless data when the number of dips is less than or equal to *k* (Figure 1*)*. This results from the fact that under these condiitions matrix $\mathbf{A}$ has at most rank *k*. The exactness property means we can consider eigen filtering even for structured data.

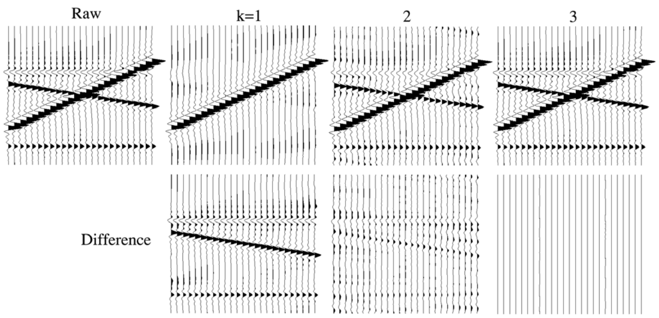

Figure 1: The exactness property. Eigen filtering is carried out with increasing values of *k*. Filtering with *k = 3* does nothing since it equals the number of distinct dips.

Efficiency

The method as described is about ten times slower than f-x prediction filtering. It can, however, be sped up using the same methods as f-xy eigen filtering (Trickett, 2002.) In particular, estimating $F_k(\mathbf{A})$ using double-truncated SVD is about three times faster than our original method with little or no difference in results.

There is an additional optimization not available to f-xy eigen filtering. The computation of $F_k(\mathbf{A})$, no matter which method is used, involves the repeated calculation of $\mathbf{Av}$ for different values of vector $\mathbf{v}$. This can be made efficient using a strategy known as "embedding in a circulant matrix" which exploits the Hankel structure of $\mathbf{A}$ and the Fast Fourier Transform (Luk and Qiao, 2000.) Regrettably it is only worthwhile when n > 30, which is usually larger than our spatial design gate.

Signal Preservation vs. Noise Suppression

The goal is to preserve as much signal as possible while removing as much noise as possible. With artificial data examples we can quantify these ideas because we know both the signal and noise. Suppose $\{F_i\}$ is the set of filtered (that is, noise suppressed) samples for all traces, $\{S_i\}$ is the set of signal samples for all traces, and $\{N_i\}$ is the set of noise samples for all traces. One measure of the percentage of signal preservation is

$$SP = 100\,\frac{\sum_i S_i F_i}{\sum_i S_i^2}.$$

100% means perfect signal preservation. 0% means there is no correlation between the result and the signal. Similarly a measure of the percentage of noise removed is

$$NR = 100\,(1 - \sqrt{\frac{\sum_i (S_i - F_i / SP)^2}{\sum_i N_i^2}}).$$

When NR = 100% the filtered data exactly matches the signal, meaning we have removed all noise. When NR = 0% the filtering has made no improvement to the noise level.

Figure 2 compares f-x eigen filtering against f-x projection (Soubaras, 1994) and f-x prediction (Canales, 1984) filtering on an artificial data set. Figure 3 charts the amount of signal preservation and noise removal in the same example. Eigen filtering matches up well with projection filtering, while prediction filtering gives the worst results.

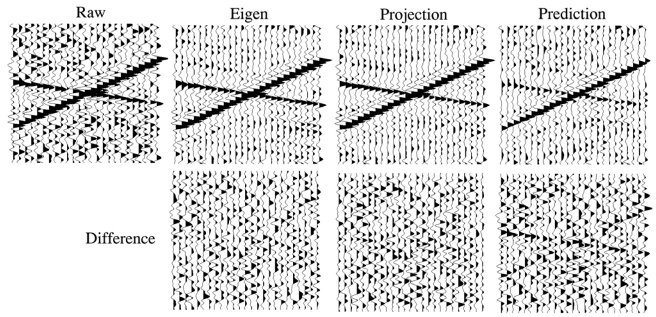

Figure 2: F-x eigen, projection, and prediction filtering on noisy artificial data. Eigen and projection filtering both show artifacts, although this does not tend to be a problem with real data.

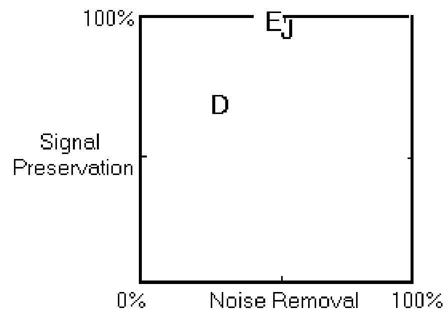

Figure 3: Estimated signal preservation vs. noise removal for Figure 2. E = eigen, J = projection, D = prediction . The top right hand corner of the graph is optimum.

Data Results and Conclusions

Figure 4 compares various filtering methods on a real structured data set. With k = 2 eigen filtering is comparable to projection filtering in this example. With k = 3 it is comparable to prediction (although it removes much less energy.) One of the advantages of eigen filtering is that by adjusting the single parameter k it can be tailored to almost any level of noise suppression.

Noise suppression along constant-frequency slices can be considered a type of spectral line analysis. The acoustics and NMR communities have developed a wealth of signal processing algorithms in this area. The geophysics community should actively mine these algorithms.

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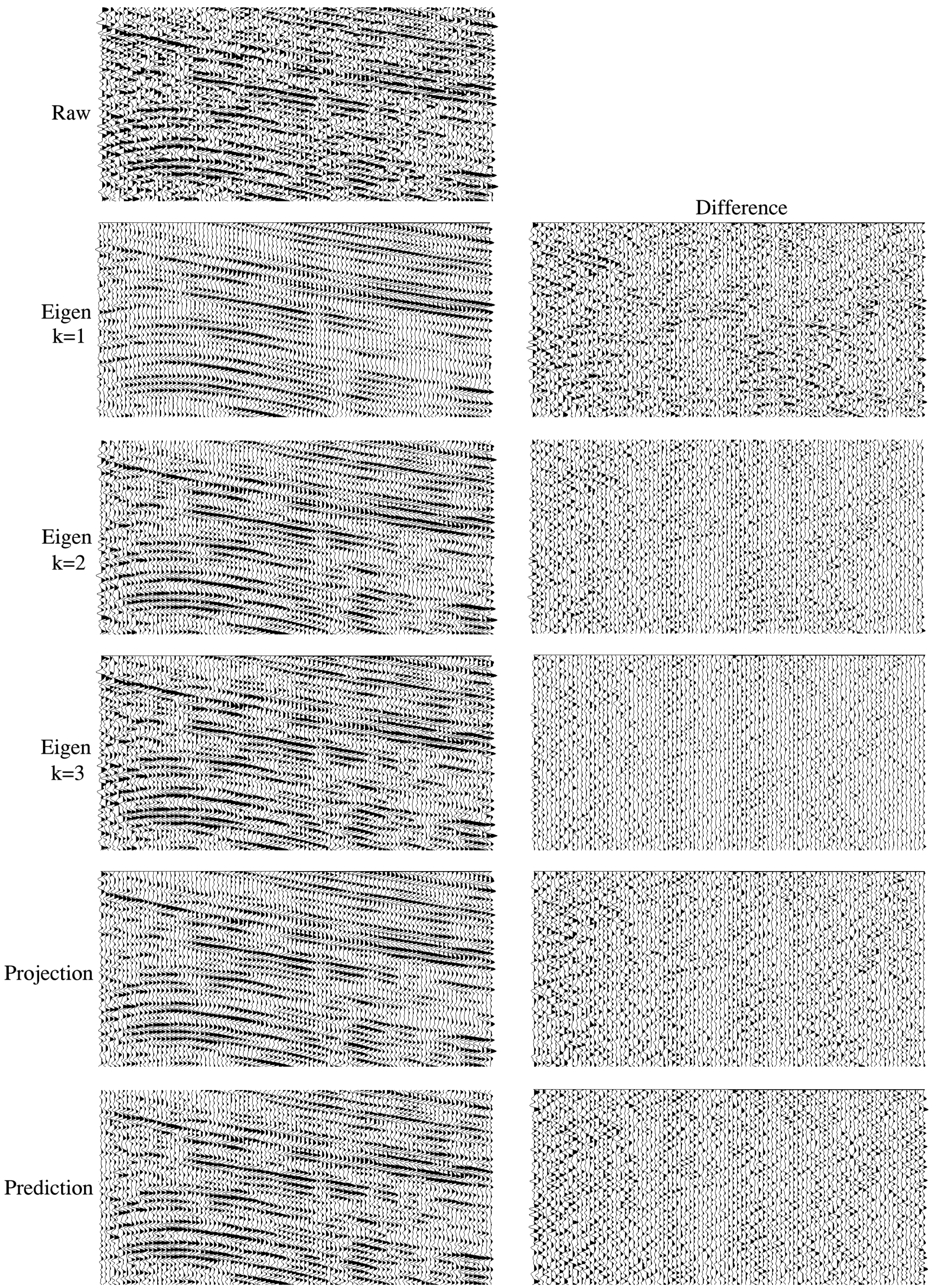

Figure 4: Noise suppression on real data using eigen, projection, and prediction filters. By adjusting *k*, eigen filtering can be tailored to almost any level of noise suppression.

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